

Własności ilorazów różnicowych

(8)

1) (i_0, i_1, \dots, i_n) dowolna permutacja $(0, 1, 2, \dots, n)$

$$f[x_{i_0}, x_{i_1}, \dots, x_{i_n}] = f[x_0, x_1, \dots, x_n]$$

np.

$$f[x_1, x_0] = \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = f[x_0, x_1]$$

Podobnie np

$$\begin{aligned} f[x_1, x_0, x_2] &= f[x_2, x_0, x_1] \\ &= f[x_0, x_1, x_2] = \dots \end{aligned}$$

$$2) f[x_0, x_0] = \lim_{x_1 \rightarrow x_0} f[x_0, x_1] = f'(x_0)$$

$$f[\underbrace{x_0, x_0, \dots, x_0}_{n \text{ razy}}] = \frac{1}{n!} f^{(n)}(x_0)$$

$$\begin{aligned} f[x_0, x_1, x_0] &= f[x_0, x_0, x_1] = \frac{f[x_0, x_1] - f[x_0, x_0]}{x_1 - x_0} \\ &= \frac{f[x_0, x_1] - f'(x_0)}{x_1 - x_0} \end{aligned}$$